Predicting the Flexural Collapse Load of Concrete Slab Bridges

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SYNOPSIS

This paper presents a new method for predicting the load which will cause a ductile reinforced concrete slab to collapse in flexure. The method is less conservative than elastic techniques, easier to use than nonlinear finite element analysis, and applicable to more general slab geometries than any plastic technique in common use. It allows engineers to verify the load-carrying capacity of existing concrete bridges and other structures, thus reducing the unnecessary strengthening or replacement of bridges currently deemed as inadequate using more conservative analysis methods.

The method is based on the lower bound theory of plasticity and uses equilibrium finite elements and optimisation by conic programming. Extensions to previous researchers’ work include the ability to calculate a rigorous lower bound to the collapse load and to predict the critical collapse mechanism. The results can be checked by hand using yield line analysis to produce a complementary upper bound and hence fully bound the ideal plastic collapse load.

The technique is applied to existing concrete slabs and is shown to produce results which are consistent with or improve upon previous analyses and experiments.

MOTIVATION

Bridge assessment

Engineers are frequently required to predict the load which would cause a reinforced concrete bridge to collapse. Typically, they seek an underestimate of the load so that the real structure can carry more load than predicted and is therefore safe.

When designing a bridge, an engineer may therefore use a conservative analysis technique to design a structure which he or she predicts to be just safe but which actually has significant extra ‘hidden’ capacity. Typically, the cost of providing this reserve capacity in a new structure is small, as it only requires minimal additional materials and labour.

However, when engineers are assessing existing bridges, it is important that they do not use unduly conservative analysis techniques. If such techniques are used, bridges which in reality have adequate strength may be deemed to be unsafe and may therefore be unnecessarily strengthened or reconstructed. In many cases, modifying an existing structure whilst it is in use may cost almost as much as, if not more than, the original construction cost. In addition, disruption to traffic or other infrastructure during strengthening may be very expensive, to the extent that such disruptions may not be economically viable. As well as being wasteful, these large
additional costs may direct limited maintenance resources away from structures which are in genuine need of repair, increasing the risk of failure associated with structures across the network.

Despite the clear need for realistic analysis of bridge load capacity, an audit of bridge assessments undertaken during the UK’s fifteen-year bridge assessment and strengthening programme found the most common cause of assessment failures of primary bridge elements to be the use of ‘conservative or inappropriate analysis’. Many assessments were conducted using linear elastic grillage or finite element methods, which predict the load causing first yield without considering any subsequent increase in load before collapse. Clearly, a less conservative method is needed.

**Alternative methods of assessment**

One alternative analysis method for assessing bridge load capacity is non-linear finite element analysis (NLFE). However, NLFE calculations are often highly sensitive to the choice of input parameters, leading to concerns over their robustness. NLFE can also be difficult to use and validate. Given that NLFE is not guaranteed to show a bridge to be safe, the high cost has tended to dissuade bridge owners from using it on small structures.

Other methods use plasticity theory. Many plastic methods are based on upper bound theory, which always gives an overestimate of the collapse load; it is left to the engineer to ensure that the overestimate is not unsafe. Some methods are also restricted to relatively simple structures. In particular, many are not able to analyse structures with realistic features such as:

- complex irregular slab geometry;
- a variety of supports conditions, including columns, simply supported edges and clamped edges;
- complex loading patterns, including sophisticated vehicle models specified by codes; and
- complex, non-uniform and non-isotropic reinforcement.

For example, the COBRAS program uses the plasticity theory of yield lines, and has been shown to give less conservative results than elastic analysis in many cases. However, the program’s upper bound nature causes engineers to doubt the safety of applying it to complex structures for which it has not been validated. The library of mechanisms automatically considered by COBRAS is limited to bridges of parallelogram plan, and it is not feasible to create such a library for fully general structure geometries to cover cases such as bridges on column supports, bifurcated approach bridges, and irregular building slab layouts.

Methods using the lower bound theory of plasticity give safe underestimates of the collapse load. However, although they can improve on elastic analysis, most give predictions which are too conservative for use in bridge assessment. For example, even with sophisticated optimisation techniques, the Hillerborg Strip method often
severely underestimates collapse loads, and it is difficult to apply to complicated structures.

The new method

A new method for safe but not unduly conservative analysis of complex structures is sought. The method proposed here uses the lower bound theory of plasticity, so it produces a safe prediction of the collapse load which is less than or equal to the ideal plastic value. However, unlike elastic analysis and other lower bound methods, it is shown to give results which are not unduly conservative. It is applicable to realistic structures with all of the complexities listed above. It could therefore be applied to structures failing more simplistic assessments, leading to savings in cost and improvements in safety.

METHOD

Figure 1 summarises the main steps in the new method. Further details are given in Jackson and Middleton 2009.

Start

Divide the slab into a triangular mesh with a quadratic moment field \((m_x, m_y, m_{xy})\) inside each element

Write linear equations to ensure that the moments in each element are in equilibrium with the applied loads and with the moments in any adjacent elements

Choose some locations within each element to be ‘control points’

Write conic inequality conditions to ensure that the moments at the control points lie within the Johansen yield surface

Use conic programming to find the moment field which maximises the applied load \(\lambda_{\text{prog}}\), subject to the equilibrium and yield conditions

Search the moment field to find points where yield is most exceeded

Is the maximum violation of yield \(R_{\text{max}}\) acceptably small?

Yes

No

Add control points at the locations where yield is most exceeded

A lower bound to the collapse load is given by \(\lambda_{\text{prog}}/R_{\text{max}}\). Yield lines can only occur in locations where yield is reached, with curvature proportional to the active point on the yield surface; these can be plotted as ‘yield line indicators’.

Figure 1: Method
Like all plastic flexural methods, this technique assumes ductile, rigid-plastic flexural behaviour, which is reasonable for lightly reinforced sections, but may be prevented by brittle concrete crushing or reinforcement rupture in some situations. It ignores membrane action, which may make the true collapse load significantly higher than predicted.

The method does not consider shear, which must be checked separately. Conversations with UK engineers suggest that many bridges, such as those with little transverse reinforcement, are shown by elastic analysis to have very low capacity in flexure. Analysis with a more advanced method (currently often COBRAS) shows a much higher flexural capacity, so that the shear capacity then becomes the limiting case and must be checked carefully.

The method uses the Johansen yield surface. Note that this criterion has been shown to be unconservative in some cases involving twisting moments. The initial calculated collapse load $\lambda_{\text{prog}}$ is an estimate of the ideal plastic value, but does not have the status of a rigorous bound. However, unlike previous methods, the identification of the maximum violation of yield in the initial solution ($R_{\text{max}}$) allows this technique to identify a rigorous lower bound to the collapse load (subject to the success of a numerical search). Choosing a tolerance on $R_{\text{max}}$ sets the desired accuracy; more accurate (less conservative) solutions take longer, although each solution in this paper was produced in a few minutes on a modern desktop computer.

Another advance over previous methods is the ability to identify the location and direction of possible yield lines. The analysis program can thus plot a field of ‘yield line indicators’ which aids identification of a critical collapse mechanism for yield line analysis. Unlike many other advanced methods, this technique can therefore be checked by conducting a simple hand yield line analysis of an approximation to the mechanism identified from the lower bound analysis. The yield line calculation gives an upper bound to the collapse load which, when combined with the lower bound result, fully bounds the exact plastic collapse load.

**AN EXPERIMENTAL EXAMPLE: COLLINS’S SLAB**

The structure shown in Figure 2 is a 1:10 scale model of a typical short-span concrete slab bridge. Like many real bridges, it has very light transverse reinforcement, making the strength highly anisotropic. Elastic analysis therefore suggests a transverse hogging failure, whereas in reality the bridge continues to carry increasing load after the slab first yields in this manner.

Collins tested the slab to failure in the laboratory at Cambridge University. She measured a collapse load some three times greater than that predicted by elastic analysis. The slab has also been analysed using COBRAS and a lower bound plastic modified Hillerborg Strip method.
Using a fine mesh, the conic programming stage of the new method initially predicts the moment field shown in Figure 3. The predicted approximate collapse load of $\lambda_{\text{prog}}=26.1\text{kN}$ lies between previous lower and upper bound analyses of the slab.

The lower bound to the collapse load obtained after several iterations to reduce $R_{\text{max}}$ is plotted in Figure 4. The result is 47% greater than the highest previous lower bound and within 2% of the lowest upper bound. The actual collapse load of the experimental model was some 25% higher due to effects such as membrane action.
Figure 4: Predictions of the collapse load of Collins’s Slab

Figure 5 compares the yield line indicators from the new method (Figure 5a) with the critical mechanism identified by COBRAS (Figure 5b). The yield line indicators correlate closely with the COBRAS mechanism, so they could be used to identify a yield line pattern and to check the collapse load manually without requiring the mechanism search performed by COBRAS.

Figure 5c shows the crack pattern observed in Collins’s original experiment. The cracks provide an indication of regions and directions of high curvature, and hence possible yield lines. There is a reasonable correlation between the predicted yield lines and the observed cracks. This gives confidence that, despite the influence of membrane action indicated by the high experimental collapse load, it is reasonable to use plasticity theory to give an estimate of the collapse load in this case.
A REAL EXAMPLE: TYNE TUNNEL ROUNDBOUT STRUCTURE

The method is now applied to a real structure. The Tyne Tunnel Roundabout Structure (Figure 6) is a reinforced concrete cellular box with a continuous slab roof supported by internal and external walls. The longitudinal and transverse top and bottom reinforcement varies across the slab. The irregular geometry, non-uniform and non-isotropic reinforcement and complex live loading patterns present challenges to many simple analysis methods.

The structure was constructed in the mid 1960s. An elastic grillage analysis conducted in the late 1990s deemed its flexural capacity to be inadequate, but in 2000 a plastic yield line analysis using the COBRAS software found the capacity to be sufficient. However, the plastic approach used had two potential shortcomings:

- The upper bound nature of the yield line analysis conducted meant that, if the failure mechanisms considered in the analysis did not include the critical collapse mode, the calculated collapse load could be unconservative.

- The method could not model the geometric complexity of the full multi-cellular structure. Each ‘cell’ of the structure was therefore modelled separately, and any potential modes involving collapse in more than one cell were neglected.

The new analysis method allows these limitations to be overcome.

In the analyses shown here, the structure is loaded with an ‘HB’ abnormal vehicle load, as specified by BD 37/01 in the UK\textsuperscript{11}.

Figure 6: Plan View of the Tyne Tunnel Roundabout Structure
Lower bound analysis

Firstly, to allow comparison with previous methods, one of the models used in the previous upper bound analysis is loaded directly into the new lower bound system. This model includes only the centre cell, and was found by COBRAS to have an upper bound collapse load factor of 1.03. Analysis with the new method finds a lower bound of 1.03; the bounds are equal to three significant figures.

Figure 7 compares the collapse mechanism from the previous upper bound analysis with the yield line indicators from the new method. As with Collins’s slab, there is a reasonable agreement between the two cases. The yield line indicators could certainly be used to identify the critical mechanism for a manual yield line calculation.

Whole-structure analysis

The new method can also model the effects of any interaction between the cells of the structure by modelling the whole slab. In the full model used here, the internal and external walls are conservatively modelled as simple line supports; in reality the walls may provide some rotational restraint to the slab. The curvature of the end walls is ignored.

The resulting yield line indicators and collapse load closely match those from the analysis of the central cell, showing that the simplified model used was adequate in this case.

Alternative load positions

Using the new model which includes the whole structure, it is possible to apply the HB abnormal vehicle load in different positions so that failure occurs in different cells. Some identified collapse mechanisms are plotted in Figure 8.
In this case, the load position considered in the upper bound analysis appears to be critical, again showing that the simplified model used was adequate.

A THEORETICAL EXAMPLE: CRUCIFORM SLAB

The method is now applied to a case from the literature which has a geometry that is unsuitable for analysis using COBRAS. It is a cruciform shaped slab with simply supported edges and a uniform unit moment capacity in hogging and sagging. Figure 9 shows the slab geometry and the yield line indicators obtained using the new method. The collapse mechanism appears to be somewhat complex. The method identified a lower bound to the collapse load of 50.0, which is within a few percent of the value previously obtained using an upper bound method\textsuperscript{12}. The upper bound method required an experienced user to identify a critical yield line mechanism, whereas the new method analyses the slab completely automatically from the geometry input.
Figure 9: Cruciform slab and yield line indicators

CONCLUSIONS

A new lower bound method to predict the collapse load of reinforced concrete slabs is presented. As long as a numerical search identifies the point of maximum yield, the method produces a rigorous lower bound to the flexural collapse load of a slab with the Johansen yield criterion.

The technique has been applied to three structures. It produces collapse load predictions which are consistently:

- less than experimentally measured flexural collapse loads (‘safe’);
- greater (less conservative) than elastic and conservative lower bound plastic results; and
- less than (consistent with) plastic upper bound results.

It predicts collapse mechanisms close to those found by upper bound methods.

The observed performance suggests that the method could be usefully applied to many slabs. It could help to demonstrate that bridges which have been deemed unsafe by elastic methods have adequate flexural strength, and hence prevent the strengthening and reconstruction currently deemed necessary by overly conservative analyses.
The good match between the new ‘safe’ lower bound and previous upper bound analyses also gives confidence in the use of upper bound methods for simple slab geometries.

REFERENCES


