A Rational Design Method for Flexible Steel Helical Culverts

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Synopsis

Measurements during backfilling of flexible helical steel culverts (Pritchard, 2008) identified high lateral earth pressures. This paper presents a theory for the development of these pressures during incremental backfilling by examining changes in horizontal and vertical soil stresses in the zones surrounding the culvert due to compaction of the backfill. During the incremental compaction process, it will be shown that the immediate layers being compacted approach the passive state. As more layers are added, the stress state in the previously compacted layers change from the passive state to the active state. The paper provides a clear description of the behaviour of the soil during compaction and explains the high lateral earth pressures that have been observed. This work has significant implications on the design loads on buried structures including flexible steel helical pipes and culverts. This paper argues that bending moment, not hoop force, is the critical design consideration. The critical design load case for maximum moment at the crown occurs when the backfill is level with the crown. Field measurements confirmed the numerical modelling of steel strain, deformations and soil pressures around a culvert.

The design method for helical culverts must include the effects of incremental backfilling. This can be undertaken using complex numerical modelling. However, the use of advanced modelling is not considered appropriate for routine design methods. This paper presents a new design method that provides a result similar to a more rigorous method without the complexity, whilst incorporating the essential features of such analysis. The new design method is based on an assumed pressure distribution that has been verified from the experimental results discussed in this paper.

After-Compaction Lateral Pressures

Lateral earth pressure on buried structures induced by applied vertical loadings has been studied as far back as Couplet (1726). Since Coulomb (1776) and Rankine (1857) presented simple theories relating to lateral and vertical pressures, a number of other more complex theories and analytical procedures have been developed by Rowe (1954), Broms (1971), Ingold (1979), and Duncan and Seed (1986). Despite these advances, compaction-induced pressures on structures continue to be a subject of intense interest.

Locked-in stresses are defined as the increment above the at rest normally consolidated stresses. It is well known that there can be considerable locked-in stresses in cohesionless fill, while in cohesive (clayey) fill, locked-in stresses dissipate in the long term. However, Duncan and Seed (1986) noted that permanent locked-in stresses have been measured in cohesive soils where the degree of saturation is less than 80% and where moisture contents are up to 2% wet of optimum. Under these conditions, locked-in stresses should be included in design. Tests carried out by Carder et al. (1977) at the TRRL Experimental Retaining Wall
Facility in England have often been used for validation by a number of researchers such as Ingold (1979) and Duncan and Seed (1986). Pritchard (2008) measured high lateral pressures similar to those reported in the literature.

During compaction, soil particles are brought into closer proximity to each other by the expulsion of air from voids. Average inter-particle stresses (effective stress) in the soil increase when compaction loads are applied. The lateral earth pressure coefficient, K, has been found to be dependent on the soil stress history. The simplest of these appears to be the one modified from Mayne and Kulhawy's relationship (1982) between the lateral earth pressure coefficient at rest, $K_o$, and the effective friction angle for a normally consolidated soil. This relationship requires an estimate of the maximum effective vertical stress experienced by the soil. The empirical expression for the lateral earth pressure at rest for an over-consolidated soil (a compacted soil), $K_{oc}$, takes the following form,

$$K_{oc} = K_o \text{ (OCR)}^n = \frac{\sigma_v'}{\sigma_v}$$  \hspace{1cm} (1)

Substituting Jáky's relationship (1944) for a normally consolidated soil of Equation 1 becomes,

$$K_{oc} = (1-\sin \phi') \text{ (OCR)}^n$$  \hspace{1cm} (2)

The effect of compaction is reflected in the over-consolidation ratio (OCR) and the exponent, n. The difficulty in estimating after-compaction stresses using Equation (2) lies in ascertaining the OCR of the soil, which is dependent on the type of compaction equipment and the method of compaction. A large number of applications of low compaction pressure does not result in the same OCR as fewer applications of larger compaction pressure. This approach is empirical and provides a simplified view of soil behaviour.

A number of methods have been proposed to estimate after-compaction pressures. Historically, most of these relate to the assumption that the lateral earth pressure coefficient is modified as a result of the compaction, resulting in over-consolidation.

![Graph showing coefficient of earth pressure vs. displacement](image_url)

**Figure 1**: Coefficient of earth pressure vs. displacement

### Soil States

Figure 1 shows the relationship between displacement and the coefficient of lateral earth pressure, K, for a specific effective friction angle, $\phi'$. The vertical axis intersects...
the displacement axis at rest position, $K_o$. The limiting active and passive pressure conditions represent the limits for compaction pressure states of the backfill material.

Figure 2 shows a horizontal plate subjecting the ground to a vertical compressive pressure. The plate represents the mechanical compaction equipment (for example, a wacker packer or a vibrating roller) used during backfilling of helical culverts. The response of the soil to this applied load depends on its location. The soil under the applied pressure compresses vertically, moving towards the active limit state. The soil material at some distance to the side of the plate heaves upwards and moves towards the passive limit state. Under this state, the effective lateral stress in an element of soil is greater than the effective vertical stress. Between these two conditions, the soil state is transitional between the active and passive states.

**Helical Culvert Response During Phases in Backfilling**

The backfilling of a helical culvert has three distinct construction phases:

- **Phase 1** – Placement of the culvert on a prepared base. A small amount of surcharge is normally placed on the culvert crown prior to commencing backfilling. This surcharge limits vertical deformation of the flexible culvert.
- **Phase 2** – Progressive placement of layers (typically about 200mm nominal thickness) from the invert until the mid-plane horizontal axis is reached.
- **Phase 3** – Progressive placement of layers above the mid-plane horizontal axis until the required cover is obtained.

Each phase has different compaction equipment.

*Phase 1 – Placement of Culvert on a Prepared Base*
The culvert is placed on the prepared base and a small amount of the fill is placed on
the crown. The surcharge on the crown of the culvert is significant during Phase 2.

Figure 3 shows the forces acting on the culvert. For vertical equilibrium:

\[ W_{\text{cul}} + W_{\text{sur}} = \int p \, dl = \text{Constant} \]  

(3)

Phase 2 – Backfilling of Culvert up to Mid-Plane Horizontal Axis

During Phase 2, the culvert is backfilled in layers on both sides up to the mid-plane
horizontal axis. The governing conditions for vertical loading in Equation (3) also
hold in Phase 2 because the vertical downward force is constant in Phases 1 and 2.
The line of action of the resultant of the contact pressure does not change.

The applied pressure distribution around the culvert adjusts when each layer is
placed; so equilibrium is maintained. The length of the cross-section in contact with
the backfill increases as each layer is placed.

Phase 3 – Backfilling of Culvert above the Mid-Plane Horizontal Axis

When the backfill is above the mid-plane horizontal axis, the forces acting on the
culvert are more complex. Issues to be considered are:

- The weight of soil above the culvert assuming no arching.
- Earth pressure from compaction.

When the backfill is above the mid-plane horizontal axis, the culvert may deform
significantly (Pritchard, 2008). The earth pressure on the culvert is sensitive to
construction variation.

Response During Backfilling

The deflected shape and co-existing bending moment derived from full scale
experiments (Pritchard, 2008) are shown in Figure 5. The experimental work showed
that once backfilling is above the mid-plane horizontal axis, the length of the vertical
axis of the culvert progressively increases until the height of fill reaches the level of
the crown.

The backfilling assumptions are:

- Plastic hinges do not form during backfilling.
- The backfill is in contact with the culvert.

Based on the fieldwork, it was observed by Pritchard (2008) that:

- Deformation of the culvert is small (typically maximum of 1.5% of the initial
diameter).
- Backfilling against the upper half of the culvert does not distort appreciably the
lower half of the culvert.
- The vertical deformation of the culvert is due to deformation in the upper half
of the culvert, particularly at the culvert crown.
- The mid-plane horizontal diameter decreases in length until backfilling reaches
the culvert crown.
- The upper half of the culvert develops bending due to the deformation (Figure
This consideration of backfilling phases shows how the development of the imposed loads on the culvert is due to the incremental nature of backfilling. Analysis methods and design codes need to consider construction sequencing. It is important to recognize that the observed response is different from the result implicit in the ring compression method currently used in Australian Standards AS 1761 (1985) and AS 1762 (1984) and other codes worldwide. These standards assume the backfill is placed instantaneously and ignores incremental backfilling and bending effects on the culvert. While the approach used in the standards represents a simplified design methodology, the underlying concept does not represent the physical behaviour of a culvert. The ring compression method can overestimate stresses resulting in overly-conservative design in some cases. In other cases, it does not prevent culvert failure during installation (Pritchard, 2008).

**Changes in Soil States in the Layer Being Compacted**

Figure 6 shows the deformed shape of the culvert during incremental backfilling at the completion of Layer $n$. Position A is a point against the culvert in Layer $n$. Two conditions will be examined that correspond to the active and passive states as defined in Figure 2.

The nominal 200mm thick backfill layers (bulk unit weight 20kN/m$^3$) have a small self-weight compared to the compaction pressure which is typically in excess of 100kPa.
Active Stress Zone – Zone Under Compaction Equipment

Backfilling involves both placement of the layer and its subsequent compaction, undertaken in a number of passes of the compaction equipment. The postulated behaviour in the active stress zone (Figure 2) is discussed below and shown in Figure 7.

![Figure 7: Horizontal and vertical stress response in active zone during backfilling](image)

The key observations are:

1. At the completion of placement of Layer \( n \) (Figure 6), it is assumed that the soil adjacent to Position A exerts an at rest pressure on the culvert. The soil has an effective friction angle \( \phi_{\text{initial}} \), assumed for this example to be typically 28 degrees. (Point (1) on Figure 7)

2. Layer \( n \) is then compacted by the application of an effective vertical pressure \( q_c \). The soil moves from the at rest stress state towards the active stress state. The effective vertical stress increases by \( q_c \). (Point (2) on Figure 7). The effective friction angle increases. Plastic and elastic deformation of the backfill occurs during compaction.

3. When the compaction equipment moves away, the effective vertical stress decreases by \( q_c \). The soil state moves along the load-unload line (Points (2) → (3) on Figure 7) towards the passive state. Elastic deformation of the soil occurs.

4. When the compaction equipment returns, the application of the compaction pressure increases the effective vertical stress by \( q_c \). The stress state moves to Point (4) on Figure 7.

5. When the compaction equipment moves away, the effective vertical stress decreases by \( q_c \). The stress state moves along the unload line to Point (5) on Figure 7 until it reaches the passive pressure limit.

6. After repeated passes of the equipment, no further change in effective friction angle and volume occurs. Under subsequent passes the stress oscillate elastically along the final load-unload line \( (i - 1) \rightarrow (i) \).

At the completion of backfilling of this layer, the initial stress state, Position (1), and the final effective stress state, Position \( (i) \), have the same effective vertical stress.
However, the effective horizontal stress state has increased. Indeed this stress moves towards and reaches the limiting passive state.

*Passive Stress Zone – Zone Beside Compaction Equipment*

Backfilling involves both placement of the layer and its subsequent compaction, undertaken in a number of passes of the compaction equipment. The postulated behaviour in the passive zone (Figure 2) is shown in Figure 8.

![Figure 8: Horizontal and vertical stress response in passive zone during backfilling](image)

**Changes in Soil States Below the Layer Being Compacted**

The final result of the compaction process, irrespective of the zone (active, transitional or passive), is that the layer being compacted has undergone an increase in lateral soil stress and the stress state moves towards the limiting passive state. This explains the high lateral earth pressure reported in Pritchard (2008) and by other researchers (Rowe, 1954; Broms, 1971; Ingold, 1979; Duncan and Seed, 1986). Each pass of the compaction equipment increases the effective friction angle of the soil. As a result of modification of soil properties from the compaction process, $\phi'$ changes from $\phi'_\text{initial}$ (typically 28 degrees) to $\phi'_\text{final}$ (approximately 38 degrees). There is an upper bound to the effective friction angle for each material for each type of construction equipment used.

After a layer has been compacted, the soil is in an over-consolidated state. Compaction of subsequent layers will have negligible effect on the physical properties (such as $\phi'$) of a previously compacted layer.

Figure 9 shows the horizontal and vertical stresses after the compaction of Layer $n$ and the corresponding stresses changes due to construction of subsequent layers. As previously discussed, the soil is in the *at rest* stress state when it is first placed. At the completion of compaction of a layer, the soil moves towards the passive stress state. In Figure 9, two cases are considered:

- The backfill has reached the passive stress limit (Case A).
- The backfill is intermediate between the *at rest* and passive stress states (Case B).
As observed in the fieldwork, when a subsequent layer is compacted, the effect on Layer \( n \) is:

- During the compaction of subsequent higher layers, the lateral earth pressure, \( \sigma_h' \), increases by less than the vertical stress at Layer \( n \).
- The effective vertical stress on Layer \( n \) increases as a result of the construction of subsequent layers of backfill. The changes in horizontal and vertical stresses result in a reduction of the ratio \( (\sigma_h' / \sigma_v') \), and the stress state moves toward the at rest stress state (Figure 9).
- There is transition in pressure from an upper limit of the passive state towards the active state (flexible culvert) or the at rest (rigid culvert) with increasing depth.
- The limiting effective lateral soil stress is the active stress state for a flexible culvert. Eventually, the stress path intercepts the limiting active state line. The backfill then follows the active stress state line (Figure 9).

In summary, it has been shown that the top layers move towards the passive state while the previously compacted lower layers move towards a lower limit of the active pressure state (flexible culverts) or at rest state (rigid culverts). These are appropriate pressure states for culvert design.

**Lateral Earth Pressure on Culvert During Incremental Backfilling**

As discussed previously, significant lateral pressures develop on the culvert when the height of fill is above the mid-plane horizontal axis.

It has been shown that the compaction of Layer \( n \) results in the backfill material moving towards the limiting passive state and the effective angle of friction increases. Hence at the completion of the compaction of Layer \( n \), the upper bound lateral earth pressure is that corresponding to the limiting passive state.
Figure 10: Variation in soil parameters with depth

Figure 10 shows the various parameters below the surface after compaction. The key features are:

- $\phi'$ – Below the mid-plane horizontal axis, it is equivalent to $\phi_{\text{initial}}'$ because access for compaction is difficult. Above the mid-plane horizontal axis, $\phi'$ increases due to effective compaction. Typically, $\phi_{\text{final}}'$ is 38 degrees.
- $\gamma_s z$ (that is, $\sigma_v'$) – increases linearly with depth.
- The lateral earth pressure curve is a function of $\phi'$ and $\gamma_s z$. The lateral earth pressure may peak at a point in the upper layers above the mid-plane horizontal axis.
- The lateral earth pressure coefficient in the upper zones exceeds the at rest state and has an upper bound of the limiting passive state.

Based on measurements of horizontal pressure at horizontal axis level (Pritchard 2008), Figure 11 shows a plot of $K$ vs. depth of backfill above the axis for pressure transducers located on the horizontal axis of a 3000mm diameter helical culvert.

The results indicate that the lateral pressure approaches the passive limit state in the top 200-300mm of backfill material. For depths of backfill greater than 500mm, the pressure state is significantly lower and for a flexible culvert approaches the limiting active soil state. Numerical modelling by Pritchard et al (2006) confirms the high lateral earth pressures in the top 300mm of backfill.

The actual pressure profile is a function of degree of compaction, material properties and culvert-soil interaction. Due to the variability of actual construction processes and sensitivity to construction sequences during backfilling, it is difficult to predict the actual response for any particular installation of a culvert. For design purposes, it is beneficial to target the upper bound loading condition.
Overview – Design Method

Many design standards for helical culverts, including AS 1762 (1984), are based on ring compression theory. Some, including Canadian Highway Bridge Design Code CAN/CSA-S6-00 (2000) make allowance for bending but the dominant effect remains ring compression. The research discussed earlier shows that bending at the crown is the critical load case and ring compression is negligible.

New Design Method

The proposed design method focuses on the maximum crown bending moment. The method consists of a basic model with an upper bound load, which produces results similar to the more complex, theoretically rigorous solution.

Design Loading

The fieldwork (Pritchard, 2008) has shown that a high passive pressure (upper bound) is present in the top 200mm of backfill and below that level transitions towards the active state. The corresponding theoretical rationale was developed in Pritchard et al (2009). The assumed pressure distribution is shown in Figure 12. The distribution recognizes the high lateral earth pressures that develop in the top layers of the backfill. The pressure distribution on the culvert detailed in Table 1 and Figure 12 are based on the distribution determined in Pritchard (2008).

![Figure 12: Pressure distribution for new design method](image)

Table 1: Lateral earth pressure distribution for all culverts

<table>
<thead>
<tr>
<th>Depth Below Crown</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 → 0.2m</td>
<td>Passive pressure (p = $K_p \gamma_s z$)</td>
</tr>
<tr>
<td>0.2m → 0.5m</td>
<td>Linear transition from $K_p \to K_a$</td>
</tr>
<tr>
<td>0.5m → 0.5D</td>
<td>Active pressure (p = $K_a \gamma_s z$)</td>
</tr>
<tr>
<td>0.5 D → 1.0 D</td>
<td>Zero pressure</td>
</tr>
</tbody>
</table>

Table 2 shows the pressure distribution with depth for typical soil parameters. The For a well-graded backfill, the recommended effective friction angle after compaction is 38 degrees. Bulk unit weight of the backfill, $\gamma_s$, is assumed as 20kN/m$^3$. 
Table 2: Pressure distribution

<table>
<thead>
<tr>
<th>Depth Below Crown</th>
<th>Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.2m</td>
<td>16.8</td>
</tr>
<tr>
<td>0.5m</td>
<td>2.4</td>
</tr>
<tr>
<td>0.5 D</td>
<td>2.4D</td>
</tr>
<tr>
<td>0.5 D → 1.0 D</td>
<td>0</td>
</tr>
</tbody>
</table>

where, D is the diameter in metres and $\gamma_s$ is 20kN/m$^2$, $\phi'$ is 38 degrees

The new design method includes an allowance for the variations inherent in all actual backfilling procedures that may not be direct inputs to the design, such as actual compaction pressures, variations in backfill layer thickness and unintentional non-symmetric loading. A sensitivity study (Pritchard, 2008) was undertaken to confirm the model (Figure 12).

Application of the method is straightforward. The pressure distribution in Figure 2 can be decomposed into a number of regular shaped elements. Figure 13 shows the force acting at the centroid of the respective elements. The resultant element horizontal force, $F_i$, acting on a ring is shown in Figure 14. The crown moment (Young, 1989), $M_c$, is

$$M_c = \sum_{i=1}^{4} \left[ \frac{-F_i D}{2\pi} \left( \pi - \theta_i \right) \left( 1 - \cos \theta_i \right) \right]$$  (5)
**Ultimate Design Bending Moment**

The ultimate limit state design method expresses the strength criterion in the following form:

\[ M_R \geq \frac{\gamma M_S}{\phi} \]  

(7)

The first yield moment is adopted as \( M_R \) for helical culverts. For this method, \( \phi \) of 0.9 and \( \gamma \) of 1.2 adopted are, which are typical values found in design codes. A low \( \gamma \) is acceptable because an upper bound loading has been adopted and the critical design situation is temporary.

**Installation Considerations**

The incremental backfilling of helical culverts results in high strains and moments at the crown of the culvert. It is not readily possible to measure strain and moment during routine construction. Monitoring vertical axis deflection is an indirect measure of strain and moment. During installation, the maximum change in vertical and horizontal diameter of the culvert should remain less than 1.5% of the initial diameter. This value is based on field observations of culverts (Pritchard, 2008) that showed satisfactory performance. During this fieldwork, a culvert with a vertical deformation of 2.3% during construction, subsequently collapsed. It is considered that the 2% recommended by AASHTO (1994) is not sufficiently conservative and should be reduced to 1.5% because it provides a factor of safety of 1.5 relative to field observations.

**Examples of Using New Design Method**

Details of the numerical modelling are presented in Pritchard (2008) for 3000mm diameter sinusoidal S125 (125mm x 25mm) profile and 1500mm diameter S68 (68mm x 13mm) sinusoidal profile culverts.

Table 3 compares the measured, numerical and new design method responses for the 3000mm diameter and the 1500mm diameter sinusoidal profile culverts. The numerical modelling and field measurements represent particular responses from the broad spectrum that might arise during construction. The new design method gives the largest moment. This is to be expected and is desirable because the new method provides a rational upper bound.

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Peak Moment at Crown (kNm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
</tr>
<tr>
<td>3000</td>
<td>3.16</td>
</tr>
<tr>
<td>1500</td>
<td>0.90</td>
</tr>
</tbody>
</table>

**Conclusions**

This paper has developed from first principles an explanation of high locked-in stresses in the top layers of the culvert backfill during compaction by considering the
soil state transitioning from \textit{at rest} state to a passive state. The effective friction angle also increases during the compaction process. This work provides a significant advancement in the understanding of induced lateral pressures during backfilling around culverts and pipes. Consideration should be given to designing buried elements for an upper bound loading equivalent to the limiting passive state in the uppermost layers of backfill.

This corresponds with high lateral pressures measured. High lateral earth pressures on culverts have also been reported in the literature and this paper. Empirical relationships, such as Jâky (1944) and Mayne and Kulhawý's relationship (1982) for over-consolidation, have provided a means of addressing the phenomenon in calculations without providing an understanding of the mechanism.

Design methods must provide for the variations present in all actual backfilling procedures (for example, non-symmetric loading and variation in compaction pressure). Because helical culverts are flexible and constructed in a soil medium, nominally similar structures may produce a range of responses in terms of hoop force, moment and deformation due to subtly different construction practices. It is important to recognize the futility of predicting a detailed response. The new design method uses an upper bound loading based on both measured results and supporting theory.

Recent research (Pritchard, 2008) has demonstrated that the critical design condition of helical steel culverts is bending during incremental backfilling when the backfill is level with the crown of the culvert. It has also been shown that negligible live load effect occurs when the culvert has sufficient cover for adequate pavement performance. This differs from the assumption of many standards worldwide, which assume the critical load case is a combination of dead load and service live load.

The new design method determines the maximum moment at the crown. The method uses effective section properties of the culvert profile. The new design method produces thicker sections than AS 1762 (1984). It is recommended that design standards be amended to represent the fundamental response of the culvert.

\textbf{Notation}

\begin{itemize}
\item \( F_i \) \hspace{1cm} Resultant horizontal force on element \( i \) due to backfill
\item \( K \) \hspace{1cm} Ratio of horizontal effective stress to vertical effective stress
\item \( K_a \) \hspace{1cm} Active lateral earth pressure coefficient
\item \( K_{\text{at}} \) \hspace{1cm} \textit{At rest} lateral earth pressure coefficient
\item \( K_{\text{oc}} \) \hspace{1cm} Over-consolidation lateral earth pressure coefficient
\item \( K_p \) \hspace{1cm} Passive lateral earth pressure coefficient
\item \( M_c \) \hspace{1cm} Bending moment at crown
\item \( M_R \) \hspace{1cm} Calculated ultimate yield resistance bending moment capacity
\item \( M_S \) \hspace{1cm} Calculated crown bending moment from pressure distribution
\item \( p \) \hspace{1cm} Ground support reaction
\item \( p_v \) \hspace{1cm} Vertical component of ground support reaction
\item \( q_c \) \hspace{1cm} Compaction pressure
\item \( W_{\text{cul}} \) \hspace{1cm} Weight of culvert
\item \( W_{\text{sur}} \) \hspace{1cm} Weight of surcharge on crown
\item \( \gamma_s \) \hspace{1cm} Bulk unit weight of soil
\item \( \theta_i \) \hspace{1cm} Included angle in radians between vertical and force
\end{itemize}
σ_h' Effective horizontal pressure
σ_v' Effective vertical pressure
φ' Effective friction angle of the soil in the normally consolidated state.
φ'_final Effective friction angle at completion of backfilling
φ'_initial Effective friction angle at commencement of backfilling

References


